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Exact solutions of stress intensity factor histories for a half plane crack in a transversely isotropic solid under transient point shear loading on the crack faces

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Abstract

Three-dimensional analysis is performed for a transversely isotropic solid containing a half plane crack subjected to point shear forces varying with time as a Heaviside function on the crack faces at a finite distance from the crack edge. The solution of this problem is treated as the superposition of two sub-problems. One considers the transient waves in an elastic half space due to the point shear loading on the surface, while the other concerns the half space with its surface subjected to such distributed shear forces that the tangential surface displacements ahead of the crack edge induced by sub-problem 1 can be canceled out. A half space subjected to a distributed dislocation on the surface is constructed as the fundamental problem, which is solved by the use of integral transforms, the Wiener–Hopf technique and the Cagniard-de Hoop method. Exact expressions are derived for the modes II and III stress intensity factors as functions of time and position along the crack edge. Some features of the solutions are discussed through numerical results.

Keywords: Transverse isotropy; Half plane crack; Transient loading; Stress intensity factor

1. Introduction

With the wide usage of macroscopically anisotropic construction materials such as geomaterials, crystals, and fiber-reinforced composites, great interest has been shown in the dynamic crack problems of anisotropic elasticity recently. For examples, Ohyoshi (1973) and Zhang and Gross (1993) considered the SH scattering of a finite crack in a transversely isotropic medium, while Dhawan (1982a,b) analyzed the interaction of a crack with incident P and SV waves. Lobanov and Novichkov (1981) investigated the diffraction of SH waves by an oblique crack in an orthotropic half plane. Norris and Achenbach (1984) studied the diffraction of P and SV waves by a semi-infinite crack in an infinite transversely isotropic material. Studies for a periodic array of cracks in transversely isotropic solids have been presented by

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Zhang (1992) for incident SH waves, and by Mandal and Ghosh (1994) for incident P waves. Transient stress intensity factors due to impact loading have been given by Kassir and Bandyopadhyay (1983) and Ang (1987) for an in-plane crack in an infinite orthotropic or transversely isotropic solid, by Shindo et al. (1986, 1992) for a crack in an orthotropic strip, by Ang (1988) for an in-plane crack in a transversely isotropic layered solid, and by Kuo (1984a,b) for an interface crack between orthotropic and fully anisotropic half planes. Rubio-Gonzalez and Mason (2000) derived an exact solution of dynamic stress intensity factors at the tip of a uniformly loaded semi-infinite crack in an orthotropic material.

Most of research works discuss two-dimensional crack problems in the literature. But perhaps, because of mathematical complexity, three-dimensional crack problems of an anisotropic medium under dynamic loading have not yet received much attention. Among the limited studies, Tsai (1982, 1988) calculated the dynamic stress intensity factors of a penny-shaped crack in transversely isotropic material due to time-harmonic elastic waves, while Kundu and Bostrom (1991, 1992) computed both the scattered far-field and COD of the crack. Lin and Keer (1989) performed the three-dimensional analysis of cracks in a layered transversely isotropic media. Mattsson et al. (1997) investigated the 3D ultrasonic crack detection in anisotropic materials. In their recent works, Zhao (2001) and Zhao and Xie (1999, 2000) obtained exact solutions of mode I problems for a half plane crack in a transversely isotropic material due to both impact loads and moving loads.

Presently, three-dimensional analysis is performed for a transversely isotropic solid containing a half plane crack, with the crack faces subjected to point shear forces varying with time as a Heaviside function at a finite distance l from the crack edge. The similar problem, but for the static case, was solved by Kachanov and Karapetian (1997) with potential theory. Nevertheless, when dynamic loading is present, the governing equations become hyperbolic ones, and the potential theory no longer applies. In this case, Laplace transforms in conjunction with the Wiener–Hopf technique prove to be powerful tools in obtaining analytic solutions. However, due to the existence of a characteristic length l in the loading function, an inconvenient exponential term (having unbounded behavior at infinity) appears when Laplace transforms are applied, which implies that the solutions of the Wiener–Hopf equation or equations are polynomials of infinite-degree. Clearly, one cannot dispose so many physical conditions to determine the unknown coefficients of such polynomials, and therefore the direct use of the Wiener–Hopf technique is inhibited. For this reason, this problem has long been considered as one that could not be solved (Freund, 1990). In his previous work (Zhao, 2001), the author proposed a methodology for dealing with the difficult, and then a scalar Wiener–Hopf problem was solved. As a continuation of the work, a vector Wiener–Hopf problem, generated from the coupling of modes II and III due to the action of shear loading, is now considered. The solution is treated as the superposition of two sub-problems. One considers the transient waves in an elastic half space generated by a point shear load varying with time as a Heaviside function on the surface, while the other concerns the half space with its surface subjected to such distributed shear forces that the tangential surface displacements ahead of the crack edge induced by sub-problem 1 can be canceled out. A half space subjected to a distributed dislocation on the surface is constructed as the fundamental problem to solve sub-problem 2. Obviously, the fundamental problem does not have a characteristic length in the loading function, and can be solved by the use of integral transforms, the Wiener–Hopf technique and the Cagniard-de Hoop method. Exact expressions are derived for the modes II and III stress intensity factors as functions of time and position along the crack edge. Some features of the solutions are discussed through numerical results.

2. Basic formulas

As shown in Fig. 1, the configuration considered is a transversely isotropic solid containing a half-plane crack. Suppose that the solid is initially stress free and at rest. A right-handed rectangular coordinate

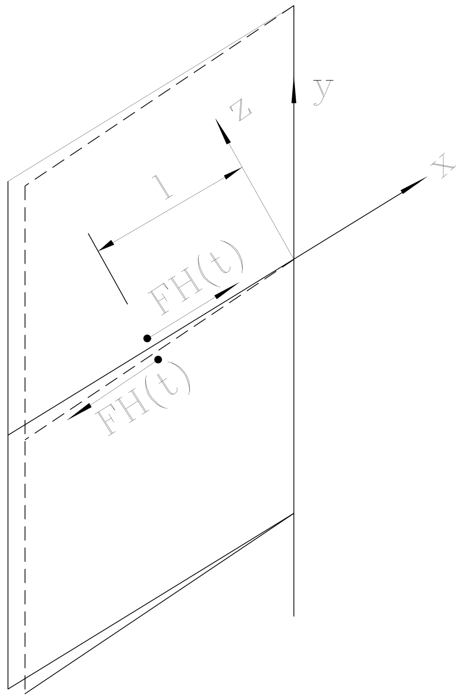

Fig. 1. A half plane crack subjected to a pair of point shear forces perpendicular to the crack edge.

system is introduced such that the y-axis coincides with the crack edge, and the half-plane crack occupies the area $z = 0$ and $x < 0$. It is also assumed that the symmetric axis of the transversely isotropic material is parallel to z-axis. At time $t = 0$ an opposed pair of point shear loads suddenly begin to act on the crack faces at a point of a finite distance l from the crack edge, resulting in a three-dimensional stress wave field in the solid.

Let $u_x(x, y, z, t)$, $u_y(x, y, z, t)$ and $u_z(x, y, z, t)$ denote the relevant displacement components in x, y and z directions respectively, then the stresses in the solid can be expressed by the relations

$$\sigma_{xx} = c_1 \frac{\partial u_x}{\partial x} + c_2 \frac{\partial u_y}{\partial y} + c_3 \frac{\partial u_z}{\partial z}, \tag{1a}$$

$$\sigma_{yy} = c_2 \frac{\partial u_x}{\partial x} + c_1 \frac{\partial u_y}{\partial y} + c_3 \frac{\partial u_z}{\partial z}, \tag{1b}$$

$$\sigma_{zz} = c_3 \frac{\partial u_x}{\partial x} + c_3 \frac{\partial u_y}{\partial y} + c_4 \frac{\partial u_z}{\partial z}, \tag{1c}$$

$$\sigma_{yz} = c_5 \left[\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right], \tag{1d}$$

$$\sigma_{xz} = c_5 \left[\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right], \tag{1e}$$

$$\sigma_{xy} = \frac{1}{2}(c_1 - c_2)\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right], \tag{1f}$$

where c_k ($k = 1$–5) are material constants.

Equations of motion for the problem are

$$\sigma_{ij,j} = \rho \ddot{u}_i \ (i = x, y, z), \tag{2}$$

where ρ is the material density.

For a transversely isotropic material it is convenient to introduce scalar potentials $\phi(x,y,z,t)$, $\psi(x,y,z,t)$ and $\theta(x,y,z,t)$, so the displacement components can be represented as (Buchwald, 1961)

$$u_x = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \tag{3a}$$

$$u_y = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}, \tag{3b}$$

$$u_z = \frac{\partial \theta}{\partial z}. \tag{3c}$$

Substitution of the above equations into Eqs. (1) and (2) gives after some manipulation

$$a_4\nabla^2\psi + a_5\frac{\partial^2\psi}{\partial z^2} = \frac{\partial^2\psi}{\partial t^2}, \tag{4a}$$

$$a_3\nabla^2\phi + a_5\nabla^2\theta + a_2\frac{\partial^2\theta}{\partial z^2} = \frac{\partial^2\theta}{\partial t^2}, \tag{4b}$$

$$a_1\nabla^2\phi + a_5\frac{\partial^2\phi}{\partial z^2} + a_3\frac{\partial^2\theta}{\partial z^2} = \frac{\partial^2\phi}{\partial t^2}, \tag{4c}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and the five constants $a_1 = c_1/\rho$, $a_2 = c_4/\rho$, $a_3 = (c_5 + c_3)/\rho$, $a_4 = (c_1 - c_2)/2\rho$, $a_5 = c_5/\rho$.

Using symmetry with respect to the plane $z = 0$, we only consider the region $z \geqslant 0$. The boundary conditions for $z = 0$ are written as

$$\sigma_{zz}(x,y,0,t) = 0, \tag{5a}$$

$$\sigma_{xz}(x,y,0,t) = -F\delta(x+l)\delta(y)H(t) + \sigma_{xz}^{+}(x,y,t), \tag{5b}$$

$$\sigma_{yz}(x,y,0,t) = \sigma_{yz}^{+}(x,y,t), \tag{5c}$$

$$u_x(x,y,0,t) = u_x^{-}(x,y,t), \tag{5d}$$

$$u_y(x,y,0,t) = u_y^{-}(x,y,t), \tag{5e}$$

where $-\infty < x,\ y < +\infty$, F is the intensity of loads, $H(\cdot)$ is the Heaviside function and $\delta(\cdot)$ is the Dirac delta function. The functions $\sigma_{xz}^{+}(x,y,t)$ and $\sigma_{yz}^{+}(x,y,t)$ represent the unknown components of stresses $\sigma_{xz}(x,y,0,t)$ and $\sigma_{yz}(x,y,0,t)$ in the region of $x \geqslant 0$, respectively; while $\sigma_{xz}^{+}(x,y,t) \equiv 0$ and $\sigma_{yz}^{+}(x,y,t) \equiv 0$ for $x < 0$. The functions $u_x^{-}(x,y,t)$ and $u_y^{-}(x,y,t)$ represent the unknown components of displacements $u_x(x,y,0,t)$ and $u_y(x,y,0,t)$ for $x < 0$, respectively; while $u_x^{-}(x,y,t) \equiv 0$ and $u_y^{-}(x,y,t) \equiv 0$ in the region of $x \geqslant 0$.

The initial conditions are expressed in terms of the potentials as

$$\phi(x,y,z,0) = \psi(x,y,z,0) = \theta(x,y,z,0) = 0, \tag{6}$$

$$\frac{\partial\phi(x,y,z,0)}{\partial t} = \frac{\partial\psi(x,y,z,0)}{\partial t} = \frac{\partial\theta(x,y,z,0)}{\partial t} = 0. \tag{7}$$

From the boundary conditions described in (5), we can see that the above formulated problem is in fact the case of a half space with its surface subjected to a point shear load varying with time as a Heaviside function in the half region of $x < 0$, while the tangential surface displacements being zero in the region of $x \geqslant 0$. Therefore, the solution of the problem can be treated as the superposition of two sub-problems. Sub-problem 1 considers a half space under the action of a point shear load on the surface, while sub-problem 2 concerns the half space subjected to shear forces $\sigma^{+}_{xz}(x,y,t)$ and $\sigma^{+}_{yz}(x,y,t)$ so that the tangential surface displacements induced by sub-problem 1 can be canceled out for $x \geqslant 0$.

The boundary conditions of sub-problem 1 can be written as

$$\sigma_{zz1}(x,y,0,t) = 0, \tag{8a}$$

$$\sigma_{xz1}(x,y,0,t) = -F\delta(x+l)\delta(y)H(t), \tag{8b}$$

$$\sigma_{yz1}(x,y,0,t) = 0. \tag{8c}$$

The solution procedures for this sub-problem are based on the use of Laplace transform over time and Hankel transforms, and the main steps are shown in the Appendix. The detailed calculation may be found in the work of Zhao (1999). If the Laplace transform of any function, say $\phi(x,y,z,t)$, is denoted by a superposed hat, that is

$$\widehat{\phi}(x,y,z,s) = \int_0^\infty \phi(x,y,z,t)\mathrm{e}^{-st}\,\mathrm{d}t, \tag{9}$$

the tangential surface displacements will take the following forms:

$$\widehat{u}_{x1}(x,y,0,s) = \frac{Fp_2}{\pi^2\rho}\int_{\frac{p_1}{p_2}}^{\infty}\left[\frac{g(v)-f(v)}{p_2^2s^2v^2}\frac{\partial^2K_0(p_2svr)}{\partial x^2} - g(v)K_0(p_2svr)\right]\mathrm{d}v, \tag{10}$$

$$\widehat{u}_{y1}(x,y,0,s) = \frac{Fp_2}{\pi^2\rho}\int_{\frac{p_1}{p_2}}^{\infty}\frac{g(v)-f(v)}{p_2^2s^2v^2}\frac{\partial^2K_0(p_2svr)}{\partial x\partial y}\mathrm{d}v, \tag{11}$$

where s is the transform parameter, $K_0(p_2srv)$ is the modified Bessel function of the second kind and

$$r = \sqrt{(x+l)^2 + y^2}, \tag{12}$$

$$g(v) = \begin{cases} 0 & v < p_3/p_2 \\ -\frac{v}{\sqrt{a_4a_5}(v^2 - a_5/a_4)^{\frac{1}{2}}} & v > p_3/p_2 \end{cases}. \tag{13}$$

$$p_1^2 = a_1^{-1}, \quad p_2^2 = a_5^{-1}, \quad p_3^2 = a_4^{-1}. \tag{14}$$

In addition, when $p_1/p_2 < v < 1$,

$$f(v) = \frac{vQ_1Q_2}{[(1-2v^2)^2 + a_5Pv^2 + a_5^2Q]^2 + (4v^2 + a_5P)^2\left(v^2 - \frac{a_5}{a_1}\right)(1-v^2)}, \tag{15}$$

$$P = \frac{4(\sqrt{a_1 a_2} - a_2)}{a_1 a_2 - (a_3 - a_5)^2}, \tag{16}$$

$$Q = \frac{P}{\sqrt{a_1 a_2}} + \frac{a_2(a_2 - a_1) + (a_3 + a_5 - a_2)(a_3 + a_2 - 3a_5)}{a_5^2[a_1 a_2 - (a_3 - a_5)^2]}, \tag{17}$$

$$Q_1 = \frac{4a_2}{p_2[a_1 a_2 - (a_3 - a_5)^2]}, \tag{18}$$

$$Q_2 = (1 - v^2)\left[1 + \frac{a_5^2}{a_1}P + a_5^2 Q + 4v^2\left(\frac{a_5}{a_1} - 1\right)\right]\beta_2 - (1 + a_5 P + a_5^2 Q)\beta_1\sqrt{v^2 - \frac{a_5}{a_1}}\sqrt{1 - v^2}, \tag{19}$$

$$\beta_1 = \left\{\sqrt{\left[\frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right] + \frac{a_1}{a_2 a_5^2}\left(v^2 - \frac{a_5}{a_1}\right)(1 - v^2)} + \frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right\}^{1/2}, \tag{20}$$

$$\beta_2 = \left\{\sqrt{\left[\frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right] + \frac{a_1}{a_2 a_5^2}\left(v^2 - \frac{a_5}{a_1}\right)(1 - v^2)} - \frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right\}^{1/2}, \tag{21}$$

$$L = a_3^2 - a_5^2 - a_1 a_2. \tag{22}$$

When $v > 1$,

$$f(v) = \frac{vQ_3\sqrt{v^2 - 1}}{(1 - 2v^2)^2 + a_5 P v^2 + a_5^2 Q - (4v^2 + a_5 P)\sqrt{v^2 - a_5/a_1}\sqrt{v^2 - 1}}, \tag{23}$$

$$Q_3 = \frac{4a_2(a_1 - a_5)}{a_1 p_2[a_1 a_2 - (a_3 - a_5)^2]}\,\frac{\beta_3 + \beta_4}{\sqrt{v^2 - a_5/a_1} + \sqrt{v^2 - 1}}, \tag{24}$$

$$\beta_3 = \left\{-\sqrt{\left[\frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right]^2 - \frac{a_1}{a_2 a_5^2}\left(v^2 - \frac{a_5}{a_1}\right)(v^2 - 1)} - \frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right\}^{1/2}, \tag{25}$$

$$\beta_4 = \left\{\sqrt{\left[\frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right]^2 - \frac{a_1}{a_2 a_5^2}\left(v^2 - \frac{a_5}{a_1}\right)(v^2 - 1)} - \frac{Lv^2 + a_2 a_5 + a_5^2}{2a_2 a_5^2}\right\}^{1/2}. \tag{26}$$

The boundary conditions of sub-problem 2 can be written as

$$\sigma_{zz2}(x, y, 0, t) = 0, \tag{27a}$$

$$\sigma_{xz2}(x, y, 0, t) = \sigma_{xz}^{+}(x, y, t), \tag{27b}$$

$$\sigma_{yz2}(x, y, 0, t) = \sigma_{yz}^{+}(x, y, t), \tag{27c}$$

$$u_{x2}(x, y, 0, t) = u_{x2}^{-}(x, y, t) - u_{x1}^{+}(x, y, t), \tag{27d}$$

$$u_{y2}(x, y, 0, t) = u_{y2}^{-}(x, y, t) - u_{y1}^{+}(x, y, t), \tag{27e}$$

where

$$u_{x1}^{+}(x,y,t)=u_{x1}(x,y,0,t)H(x), \tag{28a}$$

$$u_{y1}^{+}(x,y,t)=u_{y1}(x,y,0,t)H(x). \tag{28b}$$

Our task is to determine the functions $\sigma_{xz}^{+}(x,y,t)$, $\sigma_{yz}^{+}(x,y,t)$, $u_{x2}^{-}(x,y,t)$ and $u_{y2}^{-}(x,y,t)$, which are presented in the next two sections.

3. Required fundamental solution

As the first step of solving sub-problem 2, a fundamental problem is constructed. The problem can be viewed as a half-space problem with the material occupying the region $z \geqslant 0$, and subjected to mixed boundary conditions on $z=0$. In the Laplace transform domain, the boundary conditions are expressed as

$$\widehat{\sigma}_{zz}^{F}(x,y,0,s)=0, \tag{29a}$$

$$\widehat{\sigma}_{xz}^{F}(x,y,0,s)=\widehat{\sigma}_{xz}^{F+}(x,y,s), \tag{29b}$$

$$\widehat{\sigma}_{yz}^{F}(x,y,0,s)=\widehat{\sigma}_{yz}^{F+}(x,y,s), \tag{29c}$$

$$\widehat{u}_{x}^{F}(x,y,0,s)=\widehat{u}_{x}^{F-}(x,y,s)-\left[\frac{g(v)-f(v)}{p_2^2s^2v^2}\frac{\partial^2 K_0(p_2svr)}{\partial x^2}-g(v)K_0(p_2svr)\right]H(x), \tag{29d}$$

$$\widehat{u}_{y}^{F}(x,y,0,s)=\widehat{u}_{y}^{F-}(x,y,s)-\frac{g(v)-f(v)}{p_2^2s^2v^2}\frac{\partial^2 K_0(p_2svr)}{\partial x\,\partial y}H(x). \tag{29e}$$

The solution procedures are based on the use of transform methods and the Wiener–Hopf technique. Initially, a one-sided Laplace transform over time is applied to the partial differential Eqs. (4), taking into account initial conditions (7). Thereafter, a two-sided Laplace transform is introduced over the y coordinate. The complex transform parameter is $s\xi$, and the transformed function is denoted with a bar, for example,

$$\bar{\phi}(x,\xi,z,s)=\int_{-\infty}^{+\infty}\widehat{\phi}(x,y,z,s)e^{-s\xi y}\,dy. \tag{30}$$

Finally, a two-sided Laplace transform is used to suppress the dependence on x. The complex transform parameter is $s\eta$, and the transformed function is denoted as

$$\phi^{*}(\eta,\xi,z,s)=\int_{-\infty}^{+\infty}\bar{\phi}(x,\xi,z,s)e^{-s\eta x}\,dx. \tag{31}$$

The partial differential equations (4) are reduced to

$$-a_4s^2\mu_3^2\psi^{*}+a_5\frac{d^2\psi^{*}}{dz^2}=0, \tag{32a}$$

$$a_3s^2(\eta^2+\xi^2)\phi^{*}-a_5s^2\mu_2^2\theta^{*}+a_2\frac{d^2\theta^{*}}{dz^2}=0, \tag{32b}$$

$$-a_1 s^2 \mu_1^2 \phi^* + a_5 \frac{\mathrm{d}^2\phi^*}{\mathrm{d}z^2} + a_3 \frac{\mathrm{d}^2\theta^*}{\mathrm{d}z^2} = 0, \tag{32c}$$

where

$$\mu_1(\eta,\xi) = (p_1^2 - \eta^2 - \xi^2)^{1/2}, \tag{33}$$

$$\mu_2(\eta,\xi) = (p_2^2 - \eta^2 - \xi^2)^{1/2}, \tag{34}$$

$$\mu_3(\eta,\xi) = (p_3^2 - \eta^2 - \xi^2)^{1/2}. \tag{35}$$

The bounded solutions to Eqs. (32a)–(32c) as $z \to \infty$ may be written in the form

$$\phi^* = A\mathrm{e}^{-s\lambda_1 z} + B\mathrm{e}^{-s\lambda_2 z}, \tag{36a}$$

$$\theta^* = \frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1^2} A\mathrm{e}^{-s\lambda_1 z} + \frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2^2} B\mathrm{e}^{-s\lambda_2 z}, \tag{36b}$$

$$\psi^* = C\mathrm{e}^{-s\lambda_3 z}, \tag{36c}$$

where A, B, C are arbitrary functions of ξ and η, and

$$\lambda_{1,2}^2 = \frac{L(\eta^2+\xi^2) + a_2 + a_5}{2a_2a_5} \pm \sqrt{\left[\frac{L(\eta^2+\xi^2) + a_2 + a_5}{2a_2a_5}\right]^2 - \frac{a_1}{a_2}\mu_1^2\mu_2^2}, \tag{37}$$

$$\lambda_3 = \sqrt{\frac{a_4}{a_5}}\mu_3. \tag{38}$$

The complex η plane is cut along $\sqrt{p_1^2 - \xi^2} < |\mathrm{Re}(\eta)| < \infty$, $\mathrm{Im}(\eta) = 0$ so that $\mathrm{Re}(\mu_1) \geqslant 0$ in the entire cut η plane for each value of η, and likewise for $\mathrm{Re}(\mu_2, \mu_3, \lambda_1, \lambda_2) \geqslant 0$.

Making use of the known integral (Freund, 1990)

$$\int_{-\infty}^{+\infty} K_0(p_2 srv)\exp(-s\xi y)\,\mathrm{d}y = \frac{\pi}{s\lambda}\exp[-s(x+l)\lambda], \tag{39}$$

where

$$\lambda = \lambda(\xi) = \sqrt{p_2^2 v^2 - \xi^2}, \tag{40}$$

and substituting the transformed stresses and displacements into the boundary conditions (29), we have

$$[a_3(a_3 - a_5)(\eta^2+\xi^2) + a_2(a_1\mu_1^2 - a_5\lambda_1^2)]A + [a_3(a_3 - a_5)(\eta^2+\xi^2) + a_2(a_1\mu_1^2 - a_5\lambda_2^2)]B = 0, \tag{41a}$$

$$-\rho s^2 a_5\left[\left(\frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1} + \lambda_1\right)\eta A + \left(\frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2} + \lambda_2\right)\eta B + \xi\lambda_3 C\right] = \Sigma_{xz}^+, \tag{41b}$$

$$-\rho s^2 a_5\left[\left(\frac{a_1\mu_1^2 - a_5\lambda_1^2}{a_3\lambda_1} + \lambda_1\right)\xi A + \left(\frac{a_1\mu_1^2 - a_5\lambda_2^2}{a_3\lambda_2} + \lambda_2\right)\xi B - \eta\lambda_3 C\right] = \Sigma_{yz}^+, \tag{41c}$$

$$s\eta A + s\eta B + s\xi C = U_x^- + \frac{1}{\eta+\lambda}U_x^0, \tag{41d}$$

$$s\xi A + s\xi B - s\eta C = U_y^- + \frac{1}{\eta + \lambda} U_y^0, \tag{41e}$$

where

$$\Sigma_{xz}^+ = s \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{\sigma}_{xz}^{F+}(x, y, s) \exp[-s(\xi y + \eta x)] \,\mathrm{d}y \,\mathrm{d}x, \tag{42}$$

$$\Sigma_{yz}^+ = s \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{\sigma}_{yz}^{F+}(x, y, s) \exp[-s(\xi y + \eta x)] \,\mathrm{d}y \,\mathrm{d}x, \tag{43}$$

$$U_x^- = s^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{u}_x^{F-}(x, y, s) \exp[-s(\xi y + \eta x)] \,\mathrm{d}y \,\mathrm{d}x, \tag{44}$$

$$U_y^- = s^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \widehat{u}_y^{F-}(x, y, s) \exp[-s(\xi y + \eta x)] \,\mathrm{d}y \,\mathrm{d}x, \tag{45}$$

$$U_x^0 = -\frac{\pi\lambda[g(v) - f(v)]}{p_2^2 v^2} \exp(-sl\lambda) + \frac{\pi}{\lambda} g(v) \exp(-sl\lambda), \tag{46}$$

$$U_y^0 = \frac{\pi\xi[g(v) - f(v)]}{p_2^2 v^2} \exp(-sl\lambda). \tag{47}$$

If A, B, C are eliminated from the above equations, we will obtain

$$-\rho a_5 \mathbf{D}(\eta, \xi)\mathbf{C}(\eta, \xi)\left(\mathbf{U}^- + \frac{1}{\eta + \lambda}\mathbf{U}^0\right) = \mathbf{C}(\eta, \xi)\mathbf{\Sigma}^+, \tag{48}$$

where

$$\mathbf{U}^- = \begin{pmatrix} U_x^- \\ U_y^- \end{pmatrix}, \quad \mathbf{U}^0 = \begin{pmatrix} U_x^0 \\ U_y^0 \end{pmatrix}, \quad \mathbf{\Sigma}^+ = \begin{pmatrix} \Sigma_{xz}^+ \\ \Sigma_{yz}^+ \end{pmatrix}, \tag{49}$$

$$\mathbf{C}(\eta, \xi) = \begin{bmatrix} \eta & \xi \\ \xi & -\eta \end{bmatrix}, \tag{50}$$

$$\mathbf{D}(\eta, \xi) = \begin{bmatrix} \frac{\sqrt{a_1 a_2}}{a_2 a_5} \frac{R(\eta,\xi)}{\mu_2(\eta,\xi)} & 0 \\ 0 & \lambda_3(\eta, \xi) \end{bmatrix}, \tag{51}$$

$$R(\eta, \xi) = \frac{\left\{[(a_3 - a_5)^2 - a_1 a_2](\eta^2 + \xi^2) + a_2\right\}\mu_2 + \sqrt{a_1 a_2}\mu_1}{\sqrt{a_1 a_2}(\lambda_1 + \lambda_2)}. \tag{52}$$

Eq. (48) is a matrix Wiener–Hopf problem. In order to determine the unknown functions $\mathbf{\Sigma}^+$ and $\mathbf{U}^-$ with the Wiener–Hopf technique, we must represent any mixed function in (51) as the product of two sectionally analytic functions. To this end, we introduce a new function by defining

$$S(\eta, \xi) = \frac{R(\eta, \xi)}{k(c^2 - \eta^2 - \xi^2)}, \tag{53}$$

where $c = c_r^{-1}$, c_r is the Rayleigh wave speed of the transversely isotropic material and

$$k = \frac{(2a_2a_5)^{1/2}}{\sqrt{a_1a_2}} \frac{a_1a_2 - (a_3 - a_5)^2}{\left[-(L^2 - 4a_1a_2a_5^2)^{1/2} - L\right]^{1/2} + \left[(L^2 - 4a_1a_2a_5^2)^{1/2} - L\right]^{1/2}}. \tag{54}$$

Following the work of Zhao (2001), The function $S(\eta,\xi)$ can be decomposed into

$$S(\eta,\xi) = S_+(\eta,\xi)S_-(\eta,\xi), \tag{55}$$

where

$$S_\pm(\eta,\xi) = \exp\left\{-\frac{1}{\pi}\int_{p_1}^{p_2} \frac{[f_1(\varsigma) + f_2(\varsigma)]\varsigma d\varsigma}{\sqrt{\varsigma^2 - \xi^2}(\sqrt{\varsigma^2 - \xi^2} \pm \eta)}\right\}, \tag{56}$$

$$f_1(\varsigma) = tg^{-1}\left[\frac{(4\varsigma^2 + P)\sqrt{p_2^2 - \varsigma^2}\sqrt{\varsigma^2 - p_1^2}}{(p_2^2 - 2\varsigma^2)^2 + P\varsigma^2 + Q}\right], \tag{57}$$

$$f_2(\varsigma) = tg^{-1}\left[\frac{\beta_5\sqrt{\varsigma^2 - p_1^2} - \beta_6\sqrt{p_2^2 - \varsigma^2}}{\beta_5\sqrt{p_2^2 - \varsigma^2} + \beta_6\sqrt{\varsigma^2 - p_1^2}}\right], \tag{58}$$

$$\beta_5 = \left\{\left[\left(\frac{L\varsigma^2 + a_2 + a_5}{2a_2a_5}\right)^2 + \frac{a_1}{a_2}(\varsigma^2 - p_1^2)(p_2^2 - \varsigma^2)\right]^{1/2} + \frac{L\varsigma^2 + a_2 + a_5}{2a_2a_5}\right\}^{1/2}, \tag{59}$$

$$\beta_6 = \left\{\left[\left(\frac{L\varsigma^2 + a_2 + a_5}{2a_2a_5}\right)^2 + \frac{a_1}{a_2}(\varsigma^2 - p_1^2)(p_2^2 - \varsigma^2)\right]^{1/2} - \frac{L\varsigma^2 + a_2 + a_5}{2a_2a_5}\right\}^{1/2}. \tag{60}$$

The functions $S_+(\eta,\xi)$ and $S_-(\eta,\xi)$ are analytic and nonzero in the half planes $\mathrm{Re}(\eta) > -\sqrt{p_1^2 - \xi^2}$ and $\mathrm{Re}(\eta) < \sqrt{p_1^2 - \xi^2}$, respectively.

We also have

$$\mu_2(\eta,\xi) = \left(\sqrt{p_2^2 - \xi^2} + \eta\right)^{1/2} \cdot \left(\sqrt{p_2^2 - \xi^2} - \eta\right)^{1/2} \tag{61}$$

and

$$\lambda_3(\eta,\xi) = (a_4/a_5)^{1/4}\left(\sqrt{p_3^2 - \xi^2} + \eta\right)^{1/2} \cdot (a_4/a_5)^{1/4}\left(\sqrt{p_3^2 - \xi^2} - \eta\right)^{1/2}. \tag{62}$$

Then the matrix $\mathbf{D}(\eta,\xi)$ can be factored as

$$\mathbf{D}(\eta,\xi) = \mathbf{D}_+(\eta,\xi)\mathbf{D}_-(\eta,\xi), \tag{63}$$

where

$$\mathbf{D}_\pm(\eta,\xi) = \begin{bmatrix} \left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2} \frac{\left(\sqrt{c^2-\xi^2}\pm\eta\right)S_\pm(\eta,\xi)}{\left(\sqrt{p_2^2-\xi^2}\pm\eta\right)^{1/2}} & 0 \\ 0 & \left(\frac{a_4}{a_5}\right)^{1/4}\left(\sqrt{p_3^2 - \xi^2} \pm \eta\right)^{1/2} \end{bmatrix}. \tag{64}$$

Eq. (48) becomes

$$-\rho a_5\mathbf{D}_-(\eta,\xi)\mathbf{C}(\eta,\xi)\mathbf{U}^- - \frac{\rho a_5}{\eta+\lambda}\mathbf{D}_-(\eta,\xi)\mathbf{C}(\eta,\xi)U^0 = \mathbf{D}_+^{-1}(\eta,\xi)\mathbf{C}(\eta,\xi)\mathbf{\Sigma}^+. \tag{65}$$

The only singularity in (65) is a simple pole at $\eta = -\lambda$. This singularity can be removed by requiring the residue to be zero, so we obtain

$$\begin{aligned}&-\rho a_5\mathbf{D}_-(\eta,\xi)\mathbf{C}(\eta,\xi)\mathbf{U}^- - \frac{\rho a_5}{\eta+\lambda}[\mathbf{D}_-(\eta,\xi)-\mathbf{D}_+(\lambda,\xi)]\mathbf{C}(\eta,\xi)\mathbf{U}^0\\ &\quad = \mathbf{D}_+^{-1}(\eta,\xi)\mathbf{C}(\eta,\xi)\mathbf{\Sigma}^+ + \frac{\rho a_5}{\eta+\lambda}\mathbf{D}_+(\lambda,\xi)\mathbf{C}(\eta,\xi)\mathbf{U}^0.\end{aligned} \tag{66}$$

The right-hand side of the above equation is analytic for $\mathrm{Re}(\eta) > -\sqrt{p_1^2-\xi^2}$, and the left-hand side is analytic for $\mathrm{Re}(\eta) < \sqrt{p_1^2-\xi^2}$. Consequently, by analytic continuation, the two sides represent a single entire function vector $\mathbf{E}(\eta,\xi,s)$. At the same time, each component of $\mathbf{E}(\eta,\xi,s)$ is a constant as $\eta\to+\infty$ and has the order of $|\eta|^{1/2}$ as $\eta\to-\infty$. From the extended Liouville theorem, we have $\mathbf{E}(\eta,\xi,s)\equiv\mathbf{A}$, and $\mathbf{A}$ is a vector which is independent on the variable η. Thus,

$$\mathbf{\Sigma}^+ = \frac{\rho a_5}{\eta^2+\xi^2}\mathbf{C}(\eta,\xi)\mathbf{D}_+(\eta,\xi)\left[\mathbf{A} - \frac{1}{\eta+\lambda}\mathbf{D}_+(\lambda,\xi)\mathbf{C}(\eta,\xi)\mathbf{U}^0\right], \tag{67}$$

$$\mathbf{U}^- = -\frac{1}{\eta^2+\xi^2}\mathbf{C}(\eta,\xi)\mathbf{D}_-^{-1}(\eta,\xi)\left\{\mathbf{A} + \frac{1}{\eta+\lambda}[\mathbf{D}_-(\eta,\xi)-\mathbf{D}_+(\lambda,\xi)]\mathbf{C}(\eta,\xi)\mathbf{U}^0\right\}, \tag{68}$$

where the vector $\mathbf{A}$ remains to be determined. Let $\mathbf{A} = (A_1 A_2)^{\mathrm{T}}$ and

$$\omega = \begin{cases} -\xi & \mathrm{Im}(\xi) > 0,\\ \xi & \mathrm{Im}(\xi) \leqslant 0,\end{cases} \tag{69}$$

then it is seen from (67) and (68) that $\mathbf{\Sigma}^+$ has a simple pole at $\eta = \mathrm{i}\omega$ in $\mathrm{Re}(\eta) > -\sqrt{p_1^2-\xi^2}$, and $\mathbf{U}^-$ has a simple pole at $\eta = -\mathrm{i}\omega$ in $\mathrm{Re}(\eta) < \sqrt{p_1^2-\xi^2}$. These two poles must be removed because $\mathbf{\Sigma}^+$ and $\mathbf{U}^-$ are analytic in the half planes $\mathrm{Re}(\eta) > -\sqrt{p_1^2-\xi^2}$ and $\mathrm{Re}(\eta) < \sqrt{p_1^2-\xi^2}$, respectively. Therefore,

$$\mathbf{C}(\mathrm{i}\omega,\xi)\mathbf{D}_+(\mathrm{i}\omega,\xi)\left[\mathbf{A} - \frac{1}{\lambda+\mathrm{i}\omega}\mathbf{D}_+(\lambda,\xi)\mathbf{C}(\mathrm{i}\omega,\xi)\mathbf{U}^0\right] = \mathbf{0}, \tag{70}$$

$$\mathbf{C}(-\mathrm{i}\omega,\xi)\mathbf{D}_-^{-1}(-\mathrm{i}\omega,\xi)\left\{\mathbf{A} + \frac{1}{\lambda-\mathrm{i}\omega}[\mathbf{D}_-(-\mathrm{i}\omega,\xi)-\mathbf{D}_+(\lambda,\xi)]\mathbf{C}(-\mathrm{i}\omega,\xi)\mathbf{U}^0\right\} = \mathbf{0}. \tag{71}$$

From the above equations and the observation that $\mathbf{C}(\eta,\xi)$ has rank 1 at $\eta = \pm\mathrm{i}\omega$, we have

$$\begin{aligned}A_1 = &-\frac{\mathrm{i}\omega}{D_1^2+D_2^2}\left[\frac{D_2}{\lambda+\mathrm{i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right) - \frac{D_1}{\lambda-\mathrm{i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]U_x^0 - \frac{\xi}{D_1^2+D_2^2}\left[\frac{D_2}{\lambda+\mathrm{i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right)\right.\\ &\left.+\frac{D_1}{\lambda-\mathrm{i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]U_y^0,\end{aligned} \tag{72}$$

$$\begin{aligned}A_2 = &\frac{\xi}{D_1^2+D_2^2}\left[\frac{D_1}{\lambda+\mathrm{i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right) + \frac{D_2}{\lambda-\mathrm{i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]U_x^0 - \frac{\mathrm{i}\omega}{D_1^2+D_2^2}\left[\frac{D_1}{\lambda+\mathrm{i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right)\right.\\ &\left.-\frac{D_2}{\lambda-\mathrm{i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]U_y^0,\end{aligned} \tag{73}$$

where

$$D_1 = \left(\frac{a_2a_5}{k\sqrt{a_1a_2}}\right)^{1/2} \frac{\left(\sqrt{p_2^2-\xi^2}+\mathrm{i}\omega\right)^{1/2}}{\left(\sqrt{c^2-\xi^2}+\mathrm{i}\omega\right)S_+(\mathrm{i}\omega,\xi)}, \tag{74}$$

$$D_2 = \left(\frac{a_5}{a_4}\right)^{1/4} \frac{1}{\left(\sqrt{p_3^2-\xi^2}+\mathrm{i}\omega\right)^{1/2}}, \tag{75}$$

$$E_1 = \left(\frac{a_2a_5}{k\sqrt{a_1a_2}}\right)^{1/2} \frac{\left(\sqrt{p_2^2-\xi^2}+\lambda\right)^{1/2}}{\left(\sqrt{c^2-\xi^2}+\lambda\right)S_+(\lambda,\xi)}, \tag{76}$$

$$E_2 = \left(\frac{a_5}{a_4}\right)^{1/4} \frac{1}{\left(\sqrt{p_3^2-\xi^2}+\lambda\right)^{1/2}}. \tag{77}$$

Once $\mathbf{\Sigma}^+$ and $\mathbf{U}^-$ have been determined, the functions A, B and C can be solved from Eqs. (41). By substituting (36) into Eqs. (3) and (1) respectively, the displacements and stresses for the fundamental problem are then derived. From Eqs. (10), (11), (27) and (29), we see that the solution of sub-problem 2 can be obtained by a direct integration of the fundamental solution.

4. The stress intensity factor histories

In the field of dynamic fracture mechanics, the interest will be the determination of dynamic stress intensity factors. The stress intensity factors for the fundamental problem in the Laplace transform domain can be expressed as

$$\bar{K}_{\mathrm{II}}^F(\xi,s) = \lim_{x\to 0^+}[(2\pi x)^{1/2}\bar{\sigma}_{xz}^{F+}(x,\xi,s)], \tag{78}$$

$$\bar{K}_{\mathrm{III}}^F(\xi,s) = \lim_{x\to 0^+}[(2\pi x)^{1/2}\bar{\sigma}_{yz}^{F+}(x,\xi,s)]. \tag{79}$$

From the Abel theorem concerning asymptotic properties of transforms (Freund, 1990) and by virtue of Eqs. (42) and (43), we get

$$\begin{Bmatrix} \bar{K}_{\mathrm{II}}^F(\xi,s) \\ \bar{K}_{\mathrm{III}}^F(\xi,s) \end{Bmatrix} = \lim_{\eta\to+\infty}\left[(2s\eta)^{1/2}\mathbf{\Sigma}^+(\eta,\xi,s)\cdot s^{-1}\right]. \tag{80}$$

Substitution of (67) into the above equation yields

$$\bar{K}_{\mathrm{II}}^F(\xi,s) = \frac{\rho\pi a_5^2}{v^2}\cdot\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\sqrt{\frac{2}{s}}G_1(\xi,\omega,v)\exp(-sl\lambda), \tag{81}$$

$$\bar{K}^F_{\rm III}(\xi,s)=\frac{\rho\pi a_5^2}{v^2}\cdot\left(\frac{a_4}{a_5}\right)^{1/4}\sqrt{\frac{2}{s}}G_2(\xi,\omega,v)\exp(-sl\lambda), \tag{82}$$

where

$$G_1(\xi,\omega,v)=g_1(\xi,\omega,v)f(v)+g_2(\xi,\omega,v)g(v), \tag{83}$$

$$G_2(\xi,\omega,v)=g_3(\xi,\omega,v)f(v)+g_4(\xi,\omega,v)g(v), \tag{84}$$

$$g_1(\xi,\omega,v)=-\frac{1}{E_1}\left[\lambda+\frac{{\rm i}\omega(D_1^2-D_2^2)}{D_1^2+D_2^2}\right], \tag{85}$$

$$g_2(\xi,\omega,v)=-\frac{{\rm i}\omega v^2}{a_5\lambda(D_1^2+D_2^2)}\left[\frac{D_2}{\lambda+{\rm i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right)-\frac{D_1}{\lambda-{\rm i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]-\frac{v^2}{a_5\lambda E_1}-g_1(\xi,\omega,v), \tag{86}$$

$$g_3(\xi,\omega,v)=\frac{2\xi D_1D_2}{E_1(D_1^2+D_2^2)}, \tag{87}$$

$$g_4(\xi,\omega,v)=-\frac{v^2\xi}{a_5\lambda(D_1^2+D_2^2)}\left[\frac{D_1}{\lambda+{\rm i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right)+\frac{D_2}{\lambda-{\rm i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]-g_3(\xi,\omega,v). \tag{88}$$

The inverse two-sided Laplace transform of (81) is

$$\widehat{K}^F_{\rm II}(y,s)=\frac{\pi\rho a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\sqrt{\frac{2}{s}}\cdot\frac{s}{2\pi i}\int_{\alpha_0-{\rm i}\infty}^{\alpha_0+{\rm i}\infty}G_1(\xi,\omega,v)\exp\{-s[l\lambda(\xi)-\xi y]\}\,{\rm d}\xi, \tag{89}$$

where $y>0$ is assumed for the time being and α_0 is any real number between $-p_2v$ and p_2v. The inverse transform is carried out here through use of the Cagniard-de Hoop technique. The Cagniard contours are introduced by setting $l\lambda(\xi)-\xi y=t$, which can be solved for ξ to yield

$$\xi_\pm=-\frac{yt}{y^2+l^2}\pm\frac{{\rm i}l}{y^2+l^2}\sqrt{t^2-p_2^2v^2(y^2+l^2)}. \tag{90}$$

In the ξ plane, (90) describes a hyperbola that is denoted as $\Gamma_\pm$. When $t=t_0=p_2v\sqrt{y^2+l^2}$, the imaginary part of $\xi_\pm$ vanishes and the vertex of the hyperbola $\Gamma_\pm$ is defined by

$$\xi_0=-\frac{p_2vy}{\sqrt{y^2+l^2}}. \tag{91}$$

The Γ_+ and Γ_-, together with the inversion path of ξ and two arcs of indefinitely large radius, form a closed contour as shown in Fig. 2. Now, we shift the ξ integration to the contour. When $|\xi_0|<p_1$, it is found that the integrand in (89) is analytic inside and on this contour. Choose the appropriate branch of $\lambda(\xi)$ such that $\lambda(0)=p_2v$. Then, according to Cauchy's integral theorem and Jordan's lemma, we obtain

$$\widehat{K}^F_{\rm II}(y,s)=\frac{\rho\pi a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\sqrt{\frac{2}{s}}\cdot\frac{s}{\pi}{\rm Im}\int_{t_0}^{\infty}G_1(\xi_+,-\xi_+,v)\exp(-st)\cdot\frac{\partial\xi_+}{\partial t}\,{\rm d}t. \tag{92}$$

From the convolution theorem for Laplace transform, we have

$$K^F_{\rm II}(y,t)=\sqrt{\frac{2}{\pi}}\frac{\rho a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\frac{\partial}{\partial t}\int_{t_0}^{t}{\rm Im}\left[G_1(\xi_+,-\xi_+,v)\frac{\partial\xi_+}{\partial\tau}\right]\frac{{\rm d}\tau}{\sqrt{t-\tau}}, \tag{93}$$

where the variable t in ξ_+ should be replaced by τ.

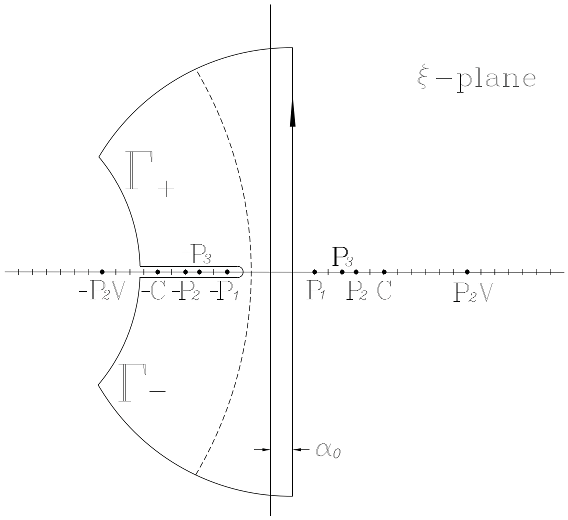

Fig. 2. The integration contour.

When $|\xi_0| > p_1$, an additional integration path from $-p_1$ to $-|\xi_0|$, which embraces the branch cut of $G_1(\xi, \omega, v)$, must be considered. Eq. (89) becomes

$$\widehat{K}^F_{\rm II}(y,s) = \frac{\rho\pi a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\sqrt{\frac{2}{s}}\cdot\frac{s}{\pi}\left\langle \operatorname{Im}\int_{t_0}^{\infty} G_1(\xi_+, -\xi_+, v)\exp(-st)\cdot\frac{\partial\xi_+}{\partial t}\,\mathrm{d}t - \operatorname{Im}\int_{p_1}^{|\xi_0|} G_1(-\xi_1, \xi_1, v)\exp\{-s[l\lambda(\xi_1) + \xi_1 y]\}\,\mathrm{d}\xi_1\right\rangle. \quad (94)$$

Let $l\lambda(\xi_1) + \xi_1 y = \tau$, then we have $t_H \leqslant \tau \leqslant t_0$, where $t_H = p_1 y + l\sqrt{p_2^2v^2 - p_1^2}$. ξ_1 is solved to be

$$\xi_1 = \frac{y\tau}{y^2 + l^2} - \frac{l}{y^2 + l^2}\sqrt{p_2^2v^2(y^2 + l^2) - \tau^2}. \quad (95)$$

The stress intensity factor in the time domain can be obtained as follows

$$K^F_{\rm II}(y,t) = \sqrt{\frac{2}{\pi}}\frac{\rho a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\frac{\partial}{\partial t}\left\{\int_{t_0}^{t}\operatorname{Im}\left[G_1(\xi_+, -\xi_+, v)\frac{\partial\xi_+}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}} - \int_{t_H}^{t_0}\operatorname{Im}\left[G_1(-\xi_1, \xi_1, v)\frac{\partial\xi_1}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}\right\}. \quad (96)$$

Combining (93) with (96), the dynamic stress intensity factor for the fundamental problem may be expressed in the form

$$K^F_{\rm II}(y,t) = \sqrt{\frac{2}{\pi}}\frac{\rho a_5^2}{v^2}\left(\frac{k\sqrt{a_1a_2}}{a_2a_5}\right)^{1/2}\frac{\partial}{\partial t}\left\{\int_{t_0}^{t}\operatorname{Im}\left[G_1(\xi_+, -\xi_+, v)\frac{\partial\xi_+}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}} - \int_{t_H}^{t_0}\operatorname{Im}\left[G_1(-\xi_1, \xi_1, v)\frac{\partial\xi_1}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}H\left(vy - \frac{p_1}{p_2}\sqrt{y^2 + l^2}\right)\right\}. \quad (97)$$

In a similar way, we have

$$K_{\mathrm{III}}^F(y,t)=\sqrt{\frac{2}{\pi}}\frac{\rho a_5^2}{v^2}\left(\frac{a_4}{a_5}\right)^{1/4}\frac{\partial}{\partial t}\left\{\int_{t_0}^{t}\mathrm{Im}\left[G_2(\xi_+,-\xi_+,v)\frac{\partial\xi_+}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}\right.$$
$$\left.-\int_{t_H}^{t_0}\mathrm{Im}\left[G_2(-\xi_1,\xi_1,v)\frac{\partial\xi_1}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}H\left(vy-\frac{p_1}{p_2}\sqrt{y^2+l^2}\right)\right\}. \tag{98}$$

With the fundamental solution and the solution of sub-problem 1 at hand, it is now possible to construct the stress intensity factor histories for the case of point shear loading varying with time as a Heaviside function on the crack faces at $x=-l$, $y=z=0$. As described in the second section, this solution can be treated as the superposition of sub-problems 1 and 2. Obviously, the shear stresses of sub-problem 1 are not singular at $x=0$, $z=0$. Therefore, the stress intensity factors are determined by sub-problem 2 only. From Eqs. (10), (11), (27) and (29), it is found that the stress intensity factors are given by

$$K_{\mathrm{II}}(y,t)=\frac{Fp_2}{\pi^2\rho}\int_{p_1/p_2}^{\infty}K_{\mathrm{II}}^F(y,t)\,\mathrm{d}v,\quad K_{\mathrm{III}}(y,t)=\frac{Fp_2}{\pi^2\rho}\int_{p_1/p_2}^{\infty}K_{\mathrm{III}}^F(y,t)\,\mathrm{d}v. \tag{99}$$

Substituting (97) and (98) into the above equations and taking into account that $t_H\leqslant t_0\leqslant t$, we finally obtain

$$K_{\mathrm{II}}(y,t)=\frac{\sqrt{2}Fa_5}{\pi^{5/2}}\left(\frac{k\sqrt{a_1a_2}}{a_2}\right)^{1/2}\frac{\partial}{\partial t}\left[\int_{p_1/p_2}^{v_0}\frac{1}{v^2}K_1(y,t,v)f(v)\,\mathrm{d}v+\int_{p_3/p_2}^{v_0}\frac{1}{v^2}K_2(y,t,v)g(v)\,\mathrm{d}v\right], \tag{100}$$

$$K_{\mathrm{III}}(y,t)=\frac{\sqrt{2}Fa_5(a_4a_5)^{1/2}}{\pi^{5/2}}\frac{\partial}{\partial t}\left[\int_{p_1/p_2}^{v_0}\frac{1}{v^2}K_3(y,t,v)f(v)dv+\int_{p_3/p_2}^{v_0}\frac{1}{v^2}K_4(y,t,v)g(v)\,\mathrm{d}v\right], \tag{101}$$

where

$$K_j(y,t,v)=\int_{t_0}^{t}\mathrm{Im}\left[g_j(\xi_+,-\xi_+,v)\frac{\partial\xi_+}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}$$
$$-\int_{t_H}^{t_0}\mathrm{Im}\left[g_j(-\xi_1,\xi_1,v)\frac{\partial\xi_1}{\partial\tau}\right]\frac{\mathrm{d}\tau}{\sqrt{t-\tau}}H\left(vy-\frac{p_1}{p_2}\sqrt{y^2+l^2}\right),\quad (j=1\text{–}4) \tag{102}$$

$$v_0=\frac{t}{p_2\sqrt{y^2+l^2}}. \tag{103}$$

Though Eqs. (100) and (101) are derived with the limitation $y>0$, they can be easily extended to the full range $-\infty<y<\infty$ by analytic continuation.

5. Case of loading parallel to the crack edge

When the point shear forces varying with time as a Heaviside function are applied in a direction parallel to the crack edge (Fig. 3), it is found that the stress intensity factors take the same forms as Eqs. (100) and (101). However, the functions $g_j(\xi,\omega,v)(j=1\text{–}4)$ should be replaced by

$$g_1(\xi,\omega,v)=\frac{\xi}{\lambda E_1}\left[\lambda+\frac{\mathrm{i}\omega(D_1^2-D_2^2)}{D_1^2+D_2^2}\right], \tag{104}$$

$$g_2(\xi,\omega,v)=-\frac{v^2\xi}{a_5\lambda(D_1^2+D_2^2)}\left[\frac{D_2}{\lambda+\mathrm{i}\omega}\left(\frac{D_1}{E_2}-\frac{D_2}{E_1}\right)+\frac{D_1}{\lambda-\mathrm{i}\omega}\left(\frac{D_2}{E_2}-\frac{D_1}{E_1}\right)\right]-g_1(\xi,\omega,v), \tag{105}$$

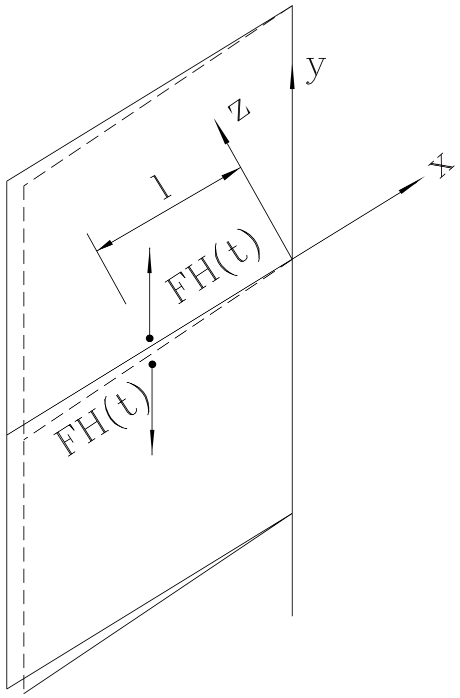

Fig. 3. A half plane crack subjected to a pair of point shear forces parallel to the crack edge.

$$g_3(\xi,\omega,v) = -\frac{2\xi^2 D_1 D_2}{\lambda E_1 (D_1^2 + D_2^2)}, \tag{106}$$

$$g_4(\xi,\omega,v) = \frac{\mathrm{i}\omega v^2}{a_5 \lambda (D_1^2 + D_2^2)} \left[\frac{D_1}{\lambda + \mathrm{i}\omega} \left(\frac{D_1}{E_2} - \frac{D_2}{E_1} \right) - \frac{D_2}{\lambda - \mathrm{i}\omega} \left(\frac{D_2}{E_2} - \frac{D_1}{E_1} \right) \right] - \frac{v^2}{a_5 \lambda E_2} - g_3(\xi,\omega,v). \tag{107}$$

6. Numerical results and discussions

We now discuss the properties of the dynamic stress intensity factor histories represented by (100) and (101). It is observed that the first term in (100) and (101) reflects the influence of the dilatational wave, shear waves and the Rayleigh wave. The function $f(v)$ has a simple pole at c/p_2, and an immediate inference is that when the Rayleigh wave arrives, the stress intensity factors will become singular at this instant. The second term in (100) and (101) is induced by shear waves alone. It is also seen that $K_j(y,t,v)$ consists of two terms. One is induced by the incident waves, and the other is due to the incident secondary waves produced by the first waves interacting with crack edge. That is to say, at a point on the crack edge, we may observe the arrivals of two kinds of waves. One kind of the waves are generated by the impact point shear loading and propagate directly towards the observing point. The other kind of the waves are also due to the impact point loading, but firstly arrive at the crack tip, and then propagate along the crack edge towards the observing point. This phenomenon is quite different from the two-dimensional case as discussed by Freund (1990).

To make the physical meaning much clear, a numerical calculation of (100) and (101) is carried out for Poisson's material that is isotropic, and for Beryl that is transversely isotropic.

Poisson's material: $a_1 = a_2 = 3a_5$, $a_3 = 2a_5$, $a_4 = a_5$, $c = 1.088/\sqrt{a_5}$.

Beryl: $a_1 = 4.12484a_5$, $a_2 = 3.61802a_5$, $a_3 = 2.01199a_5$, $a_4 = 1.17363a_5$, $c = 1.04645/\sqrt{a_5}$.

Results for $y = l$ are shown in Figs. 4 and 5, with the solid line for Poisson's material and the dashed line for Beryl. In the figures, $T = v_0p_2/p_1$, $SIF\ 2 = K_{\mathrm{II}}(y,t)(\pi l)^{3/2}/(\sqrt{2}F)$, $SIF\ 3 = K_{\mathrm{III}}(y,t)(\pi l)^{3/2}/(\sqrt{2}F)$.

It is shown in Figs. 4 and 5 that before the arrival of the dilatational wave, the medium is completely at rest and the stress intensity factors are zero. At the instant when the dilatational wave arrives, a jump takes place, and then the stress intensity factors decrease with time. Upon the arrival of the first shear wave, a discontinuity happens. When the Rayleigh wave arrives at $t = (l^2 + y^2)^{1/2} \cdot c_r^{-1}$, the stress intensity factors are of the singularity $(v_0 - c/p_2)^{-1}$ at this instant. Thereafter, the transient stress intensity factors decay gradually towards their equilibrium stress intensity factors that were obtained by Kachanov and Karapetian (1997).

This completes the analysis of a half plane crack in a transversely isotropic solid under the action of a pair of point shear loads varying with time as a Heaviside function on the crack faces at a finite distance l

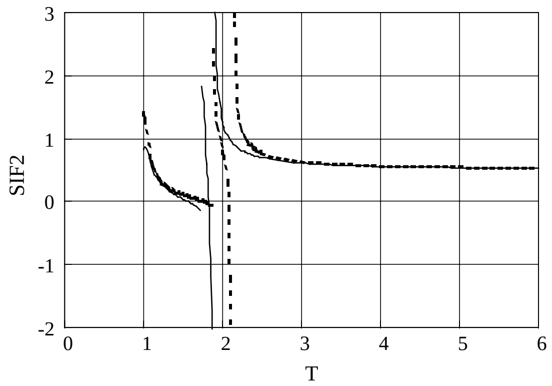

Fig. 4. The dynamic stress intensity factor history $K_{\mathrm{II}}(y,t)$.

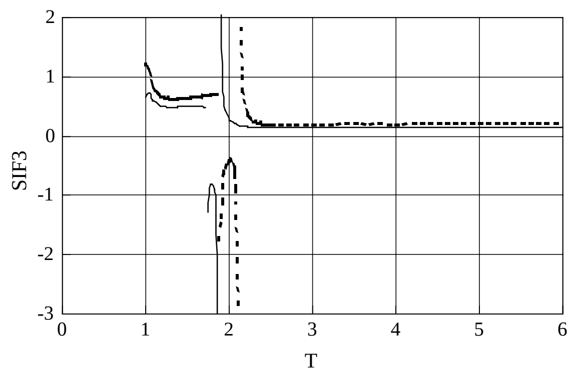

Fig. 5. The dynamic stress intensity factor history $K_{\mathrm{III}}(y,t)$.

away from the crack edge. Exact expressions are derived for the modes II and III stress intensity factors as functions of time and position along the crack edge.

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Appendix A

The dynamic response of an elastic half-space to tangential surface loading.

Consider the case that a concentrated force parallel to x-axis is applied at the origin and varies with time as a Heaviside function. In the cylindrical co-ordinate system, the boundary conditions can be described as

$$\sigma_{zz1}(r,0,t) = 0, \tag{A1.1}$$

$$\sigma_{rz1}(r,0,t) = -F\delta(r)H(t)\cos\varphi, \tag{A1.2}$$

$$\sigma_{\varphi z1}(r,0,t) = F\delta(r)H(t)\sin\varphi. \tag{A1.3}$$

For this case, the solution will take the following forms

$$u_{r1}(r,\varphi,z,t) = u_r(r,z,t)\cos\varphi, \tag{A2.1}$$

$$u_{\varphi 1}(r,\varphi,z,t) = -u_\varphi(r,z,t)\sin\varphi, \tag{A2.2}$$

$$u_{z1}(r,\varphi,z,t) = u_z(r,z,t)\cos\varphi. \tag{A2.3}$$

Further, the functions $u_r(r,z,t)$, $u_\varphi(r,z,t)$ and $u_z(r,z,t)$ can be expressed with scalar potentials $\phi(r,z,t)$, $\psi(r,z,t)$ and $\theta(r,z,t)$ as

$$u_r(r,z,t) = \frac{\partial\phi(r,z,t)}{\partial r} + \frac{1}{r}\psi(r,z,t), \tag{A3.1}$$

$$u_\varphi(r,z,t) = \frac{1}{r}\phi(r,z,t) + \frac{\partial\psi(r,z,t)}{\partial r}, \tag{A3.2}$$

$$u_z(r,z,t) = \frac{\partial\theta(r,z,t)}{\partial z}. \tag{A3.3}$$

Equations of motion become

$$a_4\left(\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{1}{r^2}\psi\right) + a_5\frac{\partial^2\psi}{\partial z^2} = \frac{\partial^2\psi}{\partial t^2}, \tag{A4.1}$$

$$a_3\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\phi\right) + a_5\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} - \frac{1}{r^2}\theta\right) + a_2\frac{\partial^2\theta}{\partial z^2} = \frac{\partial^2\theta}{\partial t^2}, \tag{A4.2}$$

$$a_1\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\phi\right) + a_5\frac{\partial^2\phi}{\partial z^2} + a_3\frac{\partial^2\theta}{\partial z^2} = \frac{\partial^2\phi}{\partial t^2}. \tag{A4.3}$$

In addition, the initial conditions of zero must be satisfied.

With the help of Laplace transform over time and Hankel transforms, the bounded solutions of Eqs. (A4.1)–(A4.3), can be obtained:

$$\widehat{\phi}(r,z,s) = \int_0^\infty [A_1 e^{-s\lambda_4 z} + B_1 e^{-s\lambda_5 z}]\xi J_1(r\xi)\,d\xi, \tag{A5.1}$$

$$\widehat{\theta}(r,z,s) = \int_0^\infty \left[\frac{(a_1\xi^2+s^2) - a_5 s^2\lambda_4^2}{a_3 s^2\lambda_4^2} A_1 e^{-s\lambda_4 z} + \frac{(a_1\xi^2+s^2) - a_5 s^2\lambda_5^2}{a_3 s^2\lambda_5^2} B_1 e^{-s\lambda_5 z}\right]\xi J_1(r\xi)\,d\xi, \tag{A5.2}$$

$$\widehat{\psi}(r,z,s) = \int_0^\infty C_1 e^{-s\lambda_6 z}\xi J_1(r\xi)\,d\xi, \tag{A5.3}$$

where A_1, B_1 and C_1 are arbitrary functions of ξ and s, and

$$\lambda_6 = \frac{1}{\sqrt{a_5}s}(a_4\xi^2 + s^2)^{1/2}, \tag{A.6}$$

λ_4 and λ_5 are the positive roots of the following equation:

$$a_2 a_5 s^4\lambda^4 - s^2[(a_5^2 - a_3^2 + a_1 a_2)\xi^2 + (a_2 + a_5)s^2]\lambda^2 + (a_1\xi^2 + s^2)(a_1\xi^2 + s^2) = 0. \tag{A.7}$$

From boundary conditions (A1.1)–(A1.3), A_1, B_1 and C_1 can be determined:

$$A_1 = -\frac{F\xi\lambda_4\lambda_5}{2\pi\rho a_5\xi^2(\lambda_5 - \lambda_4)} \frac{(a_1 a_2\xi^2 + a_2 s^2) - a_2 a_5 s^2\lambda_5^2 - a_3(a_3 - a_5)\xi^2}{(a_1\xi^2 + s^2)s^2 + s^2\{[a_1 a_2 - (a_3 - a_5)^2]\xi^2 + a_2 s^2\}\lambda_4\lambda_5}, \tag{A8.1}$$

$$B_1 = \frac{F\xi\lambda_4\lambda_5}{2\pi\rho a_5\xi^2(\lambda_5 - \lambda_4)} \frac{(a_1 a_2\xi^2 + a_2 s^2) - a_2 a_5 s^2\lambda_4^2 - a_3(a_3 - a_5)\xi^2}{(a_1\xi^2 + s^2)s^2 + s^2\{[a_1 a_2 - (a_3 - a_5)^2]\xi^2 + a_2 s^2\}\lambda_4\lambda_5}, \tag{A8.2}$$

$$C_1 = \frac{F\xi}{2\pi\rho a_5 s^2\xi^2\lambda_6}. \tag{A8.3}$$

Then, we have

$$\widehat{u}_r(r,0,s) = \frac{Fa_2}{4\pi\rho\sqrt{a_1 a_2}}\int_0^\infty \frac{\xi\sqrt{\xi^2 + p_2^2}}{R_1(\xi)}[J_0(sr\xi) - J_2(sr\xi)]\,d\xi + \frac{F}{4\pi\rho\sqrt{a_4 a_5}}\int_0^\infty \frac{\xi}{\sqrt{\xi^2 + p_3^2}}[J_0(sr\xi) + J_2(sr\xi)]\,d\xi, \tag{A.9}$$

$$\widehat{u}_\varphi(r,0,s) = \frac{Fa_2}{4\pi\rho\sqrt{a_1 a_2}}\int_0^\infty \frac{\xi\sqrt{\xi^2 + p_2^2}}{R_1(\xi)}[J_0(sr\xi) + J_2(sr\xi)]\,d\xi + \frac{F}{4\pi\rho\sqrt{a_4 a_5}}\int_0^\infty \frac{\xi}{\sqrt{\xi^2 + p_3^2}}[J_0(sr\xi) - J_2(sr\xi)]\,d\xi, \tag{A.10}$$

where

$$R_1(\xi) = \frac{\{[a_1 a_2 - (a_3 - a_5)^2]\xi^2 + a_2\}\sqrt{\xi^2 + p_2^2} + \sqrt{a_1 a_2(\xi^2 + p_1^2)}}{\sqrt{a_1 a_2}(\lambda_7 + \lambda_8)}, \tag{A.11}$$

$$\lambda_{7,8}^2 = \frac{L\xi^2 + a_2 + a_5}{2a_2a_5} \pm \sqrt{\left[\frac{L\xi^2 + a_2 + a_5}{2a_2a_5}\right] - \frac{a_1}{a_2}(\xi^2 + p_1^2)(\xi^2 + p_2^2)}. \tag{A.12}$$

Introducing the contour integration similar to the solution of Lamb's problem for the isotropic material (Eringen and Suhubi, 1975), we further have

$$\begin{aligned}\widehat{u}_r(r,0,s) = {} & \frac{F}{2\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} - \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]K_2(srv)\,\mathrm{d}v \\ & - \frac{F}{2\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} + \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]K_0(srv)\,\mathrm{d}v,\end{aligned} \tag{A.13}$$

$$\begin{aligned}\widehat{u}_\varphi(r,0,s) = {} & -\frac{F}{2\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} - \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]K_2(srv)\,\mathrm{d}v \\ & - \frac{F}{2\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} + \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]K_0(srv)\,\mathrm{d}v.\end{aligned} \tag{A.14}$$

The surface displacements in x and y directions take the following forms:

$$\widehat{u}_{x1} = \widehat{u}_r(r,\varphi,0,s)\cos\varphi - \widehat{u}_\varphi(r,\varphi,0,s)\sin\varphi, \tag{A.15}$$

$$\widehat{u}_{y_1} = \widehat{u}_r(r,\varphi,0,s)\sin\varphi + \widehat{u}_\varphi(r,\varphi,0,s)\cos\varphi. \tag{A.16}$$

By using the relations

$$\sin\varphi = \frac{y}{r}, \quad \cos\varphi = \frac{x}{r}, \tag{A.17}$$

$$\left(\frac{2x^2}{r^2} - 1\right)K_2(srv) = \frac{2}{s^2v^2}\frac{\partial^2 K_0(srv)}{\partial x^2} - K_0(srv), \tag{A.18}$$

$$\frac{xy}{r^2}K_2(srv) = \frac{1}{s^2v^2}\frac{\partial^2 K_0(srv)}{\partial x\partial y}, \tag{A.19}$$

we finally have

$$\begin{aligned}\widehat{u}_{x1} = {} & \frac{F}{\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} - \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]\frac{1}{s^2v^2}\frac{\partial^2 K_0(srv)}{\partial x^2}\,\mathrm{d}v \\ & - \frac{F}{\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}}\right]K_0(srv)\,\mathrm{d}v,\end{aligned} \tag{A.20}$$

$$\widehat{u}_{y1} = \frac{F}{\pi^2\rho}\int_0^\infty \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}} - \frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right]\frac{1}{s^2v^2}\frac{\partial^2 K_0(srv)}{\partial x\partial y}\,\mathrm{d}v. \tag{A.21}$$

Let

$$f(v) = \mathrm{Im}\left[\frac{a_2v\sqrt{p_2^2 - v^2}}{\sqrt{a_1a_2}R_1(\mathrm{i}v)}\right], \quad g(v) = \mathrm{Im}\left[\frac{v}{\sqrt{a_4a_5}\sqrt{p_3^2 - v^2}}\right], \tag{A.22}$$

and change the integration variable v to p_2v, we obtain Eqs. (10) and (11).

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